ON DIFFUSION INTERACTION OF SOLID PARTICLES
AT HIGH PÉCLET NUMBERS

D. D. POLIANIN
(Moscow)

The three-dimensional problem of steady convective diffusion to surfaces of solid particles in a laminar stream of viscous incompressible fluid is analyzed by the method of joining asymptotic expansions (in high Péclet numbers). The stream velocity field is assumed known from the solution of the related hydrodynamic problem. A similar analysis was carried out in [1, 2] in the case of axisymmetric particles located along the stream axis.

It is shown that the stress contains arrays of particles which are free of diffusion effects between each other, and in which the concentration distribution and the total diffusion flux to a particle are determined by diffusion to particles upstream of such arrays. Formulas for concentration distribution and total diffusion flux on the surface of each particle are obtained for the case when the distance between particles in the array is considerably smaller than $aP^{1/3}$ (P is the Péclet number and a is a characteristic dimension of particles). In arrays with periodic flow field structure the total diffusion flux on a particle is

$$I_k = I_1 [k^{2/3} - (k-1)^{2/3}]$$

where I_1 is the total flux on the first particle and k is the ordinal number of a particle in the array.

A qualitative analysis of the dependence of mass exchange between solid phase and fluid in concentrated disperse systems on the Reynolds number is carried out, and the plane problem of diffusion to an array of cylinders is considered.

1. Statement of the problem. We consider the three-dimensional problem of convective diffusion at the surface of solid particles in a laminar stream of a viscous incompressible fluid, and assume that a unique normal can be drawn at any point of the surface of each particle and that there exists a region in which such normals do not intersect.

The stagnation point of the particle surface (i.e. the point approached by a stream-line) in whose vicinity the normal velocity component is directed toward (away from) the surface is called the flow-on (flow-off) point, and the streamline connected to that point, the flow-on (flow-off) trajectory. We shall call the flow-on and flow-off trajectories the carrying lines. The stagnation points may be isolated or form stagnation lines on the particle surface.

For the time being we assume that the surface of each particle has only two isolated stagnation points. By the law of mass conservation one of these is the flow-on and the other the flow-off point. The set of particles and of carrying lines emerging from these is called the array, when any points of such array can be connected by a continuous curve which passes over particle surfaces and carrying lines (Fig. 1). Particles are

consecutively numbered beginning from the extreme particle first reached by a flow-on trajectory coming from infinity.

We assume that fluid flow field was determined by solving the related hydrodyna-mic problem and introduce for each array a local orthogonal system
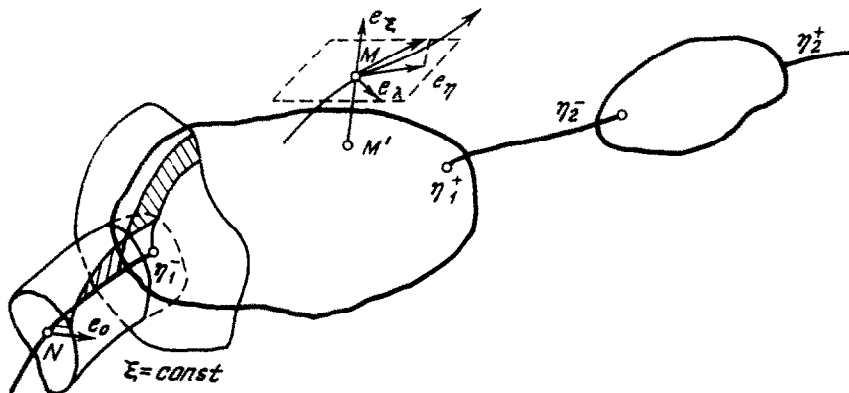

Fig. 1

of coordinates ξ, η, λ. We must indicate the direction of unit vectors at any point M lying close to the array and, also, the manner of reading curvilinear coordinates.

Point M' of the array which is closest to point M determines the direction of the unit vector e_ξ and the segment $|MM'|$ determines the dimensionless coor-dinate ξ (reduced with respect to the characteristic dimension of a particle, which is assumed common for the whole array). Since the carrying lines originate at the part-icle surface, the introduced coordinate surface $\xi =$ const are not smooth. Because of this we provide means for local smoothing.

Let us consider the simplest case when the carrying line reaching the particle is normal to its surface. The coordinate surfaces $\xi =$ const in the stagnation point neighborhood with the x-axis directed along the carrying line and the y-axis in the plane tangent to the particle surface at that point, are shown in Fig. 2. For a fixed ξ of curve

$$[x - \xi + R(\xi)]^{2n} + [y - \xi - R(\xi)]^{2n} = R^{2n}(\xi)$$

$$x \leqslant \xi + R(\xi), \quad y \leqslant \xi + R(\xi)$$

$$R(\xi) = \begin{cases} a, & \xi \geqslant \xi_0 \\ a\xi\xi_0^{-1}, & \xi < \xi_0 \end{cases} \quad (a > 0, \ \xi_0 > 1)$$

it is possible to smooth out the coordinate surfaces $\xi =$ const to any required smooth-ness (depending on n). Hence we assume henceforth that these coordinate surfaces are smoothed out by a similar procedure.

The direction of unit vector e_η is determined by the projection of the fluid vel-ocity vector at point M on the plane normal to e_ξ and the unit vector e_λ is chosen so that the system of vectors e_ξ, e_η, e_λ is an orthogonal right-hand coordinate tri-hedral. We fix the coordinate plane, where the second curvilinear coordinante (η) has a constant value, at point N which lies on the flow-on trajectory of the first particle. In the plane tangent to that surface at point N we arbitrarily fix vector e_0.
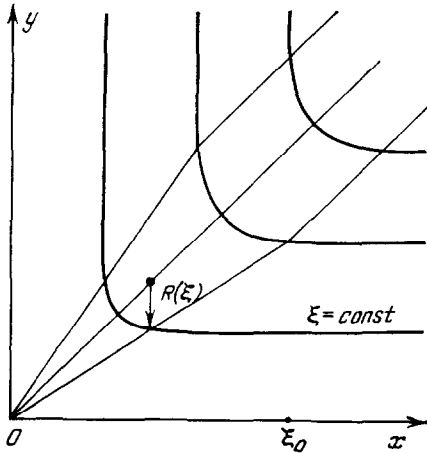
Fig. 2

The direction of the carrying line and of that vector determine the coordinate surface $\lambda = 0$ which will be used as the reference plane. The quantity λ is determined by the angle between e_0 and the vector normal to the coordinate surface $\lambda = \text{const}$ at point N $(0 \leqslant \lambda \leqslant 2\pi)$. The coordinate η is the length of arc measured from point N along the line of intersection between surface $\lambda = 0$ and the surface of the array $(\xi = 0)$ (Fig. 1).

In that system of coordinates the fluid velocity vector is at every point of the form $\mathbf{u} = \{u_\xi, u_\eta, 0\}$, and near the array (with $\xi \to 0$) has the following properties:

$$u_\xi = \xi^2 O(1), \quad u_\eta = \xi O(1) \tag{1.1}$$

$$u_\xi = \xi O(1), \quad u_\eta = O(1) \tag{1.2}$$

The first of these properties holds near the particle surface and follows from the hydrodynamic boundary condition of sticking, while the second obtains close to carrying lines, since the directional vector of these trajectories coincides with the direction of the fluid velocity vector. These conditions do not hold in proximity of stagnation points. The components of the metric tensor for $\xi \to 0$) have the following properties:

$$g_{11} = O(1), \quad g_{22} = O(1), \quad g_{33} = O(1) \tag{1.3}$$

$$g_{11} = O(1), \quad g_{22} = O(1), \quad g_{33} = \xi^2 O(1) \tag{1.4}$$

of which (1.3) obtains in the vicinity of particle surfaces and (1.4) in that of carrying lines.

Below we assume that the related flow field is quasi-stationary and that in the considered time interval particles belong to the same array (this is so when the system of particles is stationary (fixed) and the fluid stream is steady).

Taking the above into consideration, neglecting the derivatives with respect to time, and assuming total absorption of the diffusing substance on the surface of particles and constant concentration away from the latter, for the steady convective diffusion we obtain the dimensionless equation

$$\frac{u_\xi}{\sqrt{g_{11}}} \frac{\partial c}{\partial \xi} + \frac{u_\eta}{\sqrt{g_{22}}} \frac{\partial c}{\partial \eta} = \tag{1.5}$$

$$\frac{\varepsilon^3}{\sqrt{g}} \left\{ \frac{\partial}{\partial \xi} \frac{\sqrt{g}}{g_{11}} \frac{\partial c}{\partial \xi} + \frac{\partial}{\partial \eta} \frac{\sqrt{g}}{g_{22}} \frac{\partial c}{\partial \eta} + \frac{\partial}{\partial \lambda} \frac{\sqrt{g}}{g_{33}} \frac{\partial c}{\partial \lambda} \right\}$$

$$c \mid_{\xi \to 0, \, \eta_k^- \leqslant \eta \leqslant \eta_k^+} = 0, \quad c \mid_{\xi \to \infty} \to 1 \quad (k = 1, 2, \ldots, n)$$

$$g = g_{11} g_{22} g_{33}, \quad \varepsilon^{-3} = P = aU / D$$

where P is the Péclet number, a is the characteristic dimension of particles, U is the characteristic velocity of the oncoming stream, D is the diffusion coefficient, and η_k^- (η_k^+) is the coordinate of point of flow-on (flow-off) of the k-th particle.

2. Diffusion to a single particle. The equation of continuity for an incompressible fluid is of the form

$$(\nabla \cdot \mathbf{u}) = \frac{1}{\sqrt{g}} \left[\frac{\partial}{\partial \xi} \left(u_\xi \sqrt{\frac{g}{g_{11}}} \right) + \frac{\partial}{\partial \eta} \left(u_\eta \sqrt{\frac{g}{g_{22}}} \right) \right] = 0 \qquad (2.1)$$

We determine function $\Phi (\xi, \eta, \lambda)$ as the solution of equation

$$\frac{\partial}{\partial \xi} \Phi = \sqrt{\frac{g}{g_{22}}} u_\eta, \quad \frac{\partial}{\partial \eta} \Phi = -\sqrt{\frac{g}{g_{11}}} u_\xi, \quad \Phi (0, 0, \lambda) = 0 \qquad (2.2)$$

The equation of continuity is then automatically satisfied. System (2.2) has a unique solution, since the integrability condition is satisfied by virtue of (2.1) [3]. Function Φ vanishes at the array surface.

For a homogeneous and rectilinear stream at infinity the surfaces $\Phi (\xi, \eta, \lambda) =$ const have a simple physical meaning. The flow-on trajectory upstream of the first particle is in that case a straight line. Separated elements of fluid lying at equal distances from that line away from the particle provide a good representation of surface $\Phi = $ const by following these in their flow past the array. When the flow field is axially symmetric, Φ represents the conventional stream function.

It can be shown that surface $\Phi (\xi, \lambda, \eta) = $ const is formed by streamlines, and the intersection of surfaces $\Phi = $ const and $\lambda = $ const separated out the streamline.

With allowance for (2.2) problem (1.5) assumes the form

$$-\frac{\partial (c, \Phi)}{\partial (\xi, \eta)} = \varepsilon^3 \left\{ \frac{\partial}{\partial \xi} \frac{\sqrt{g}}{g_{11}} \frac{\partial c}{\partial \xi} + \frac{\partial}{\partial \eta} \frac{\sqrt{g}}{g_{22}} \frac{\partial c}{\partial \eta} + \frac{\partial}{\partial \lambda} \frac{\sqrt{g}}{g_{33}} \frac{\partial c}{\partial \lambda} \right\} \qquad (2.3)$$

$$c \mid_{\xi \to 0, \, \eta_k^- \leqslant \eta \leqslant \eta_k^+} = 0, \quad c \mid_{\xi \to \infty} \to 1 \quad (k = 1, 2, \ldots, n)$$

Properties (1.1) $-$ (1.4) imply that function Φ in proximity of the array ($\xi \to 0$) can be represented in the form

$$\Phi (\xi, \eta, \lambda) = \xi^2 f (\eta, \lambda) \qquad (2.4)$$

$$f (\eta, \lambda) \mid_{\eta \to \eta_k^-} \to \alpha_k^- (\lambda) (\eta - \eta_k^-)^2$$

$$f (\eta, \lambda) \mid_{\eta \to \eta_k^+} \to \alpha_k^+ (\lambda) (\eta - \eta_k^+)^2$$

In what follows we assume that the Péclet number is high, i.e. $\varepsilon \ll 1$.

Asymptotic analysis of Eq. (2.3) with allowance for (2.4) shows that when $\varepsilon \ll 1$ it is possible to separate in the neighborhood of a particle several regions with different mass transfer mechanisms, as schematically shown in Fig. 3. These regions are the external region e, the region b of the leading critical point, the diffusive boundary layer d, and the region of the diffusion wake W which itself consists of subregions $W^{(k)}$ ($k = 1, 2, 3, 4$).

In each of these regions Eq. (2.1) is replaced by an approximate one by separating the principal terms of expansion in the small parameter ε.
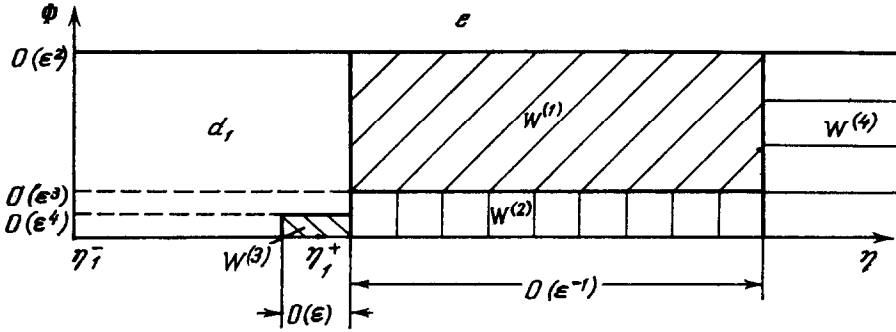

Fig. 3

The correspondence of solutions in individual regions is established by asymptotic joining.

In the external region
$$e = \{O\,(\varepsilon) < \xi, \quad O\,(\varepsilon) < |\,\eta - \eta_k^+\,|, \quad O\,(\varepsilon) < |\,\eta - \eta_k^-\,|\}$$
(here and in what follows the inequalities in braces indicate the order of characteristic dimensions of the considered region, where the interval $0 \leqslant \lambda \leqslant 2\pi$ of variation of parameter λ is omitted) the right-hand side of Eq. (2.3) is immaterial. Hence the concentration of the dissolved component in e is constant and equal unity.

Mass exchange in the boundary layer
$$d_1 = \{\Phi < O\,(\varepsilon^2), O\,(\varepsilon) < \eta - \eta_1^-, \quad O\,(\varepsilon) < \eta_1^+ - \eta\}$$
is characterized by that the diffusion transfer of substance along the surface can be neglected since it is small in comparison with the transfer along the normal to it. We introduce the variables $\zeta = \varepsilon^{-1}\Phi^{1/2}$, η, λ and from (2.3) with allowance for (2.4) for the determination of concentration in d_1 obtain the following problem:

$$L\,(t_1, \zeta)\,c_1^{(d)} = 0, \quad L = \frac{\partial}{\partial t} - \zeta^{-1}\frac{\partial^2}{\partial \zeta^2} \tag{2.5}$$

$$c_1^{(d)}\big|_{t_1=0} = 1, \quad c_1^{(d)}\big|_{\zeta=0} = 0, \quad c_1^{(d)}\big|_{\zeta\to\infty} \to 1$$

$$t_1 = \tau\,(\eta, \eta_1^-, \lambda), \quad \tau = (\eta, \mu, \lambda) = \frac{1}{2}\int_\mu^\eta f^{1/2}\,(\eta, \lambda)\,\Lambda\,(\eta, \lambda)\,d\eta$$

$$\Lambda\,(\eta, \lambda) = [\sqrt{g}\,g_{11}^{-1}]_{\xi=0}$$

For which the initial condition relates to the condition of flow-on at the stagnation point. The solution of problem (2.5) is of the form

$$c_1^{(d)}\,(\zeta,\,t_1) = \Gamma^{-1}\,(^1/_3)\,\gamma\,(^1/_3,\,\zeta^3\,/\,9t_1) \tag{2.6}$$

where $\gamma\,(^1/_3,\,x)$ is an incomplete gamma function.

The local and total diffusion fluxes at the particle surface are determined by formulas

$$j\,(\eta,\,\lambda) = \frac{1}{\sqrt{g_{11}^0}}\,\frac{\partial c_1^{(d)}}{\partial \xi}\bigg|_{\xi=0} = \frac{f^{1/2}\,(\eta,\,\lambda)}{\varepsilon\,\sqrt{g_{11}^0}\,\Gamma\,(^1/_3)}\,\frac{3^{1/3}}{t_1^{1/3}\,(\eta,\,\lambda)}\,, \quad g_{11}^0 = [g_{11}]_{\xi=0} \tag{2.7}$$

$$I = \int_s j\,ds = \int_0^{2\pi} \int_{\eta_1^-}^{\eta_1^+} j\,(\eta,\,\lambda)\,\Lambda\,(\eta,\,\lambda)\,\sqrt{g_{11}{}^0}\,d\eta\,d\lambda$$

Analysis of the rejected and retained terms of Eq. (2.3) shows that solution (2.6) is invalid when $\eta \to \eta_1^+$. Hence it is necessary to consider here, as well as in the case of axisymmetric flow around the particle [1], the diffusion wake

$$W_1 = \{\Phi < O\,(\varepsilon^2),\ \eta_1^+ - \varepsilon < \eta\}$$

which consists of four subregions $W_1^{(k)}$ ($k = 1, 2, 3, 4$), as shown in Fig. 3.

In the convective boundary layer region

$$W_1^{(1)} = \{\Phi < O\,(\varepsilon^2),\ O\,(\varepsilon) < \eta - \eta_1^+\}$$

the right-hand side of Eq. (1.5) is immaterial, hence the concentration depends here only on Φ and λ, and is constant and equal to that at exit from the diffusion boundary layer.

The explicit expression for concentration in $W_1^{(1)}$ is determined by joining with solution (2.6) and is of the form

$$c_1^{(1)}\,(\zeta,\,\lambda) = c_1^{(d)}\,(\zeta,\,\eta,\,\lambda)_{\eta \to \eta_1^+} = \Gamma^{-1}\,(^1/_3)\,\gamma\,(^1/_3,\ \zeta^3\,/\,9t_1^0\,(\lambda)) \qquad (2.8)$$

$$t_1^0\,(\lambda) = t_1\,(\eta,\,\lambda)_{\eta = \eta_1^+}$$

Region of the trailing stagnation point $W_1^{(3)} = \{\Phi < O\,(\varepsilon^4),\ |\,\eta - \eta_1^+\,| < O\,(\varepsilon)\}$ where both, the normal and the tangential transfer are significant, the inner region of the wake $W_1^{(2)} = \{\Phi < O\,(\varepsilon^3),\ O\,(\varepsilon) < \eta - \eta_1^+ < O\,(\varepsilon^{-1})\}$, and the mixing region $W_4^{(4)} = \{\Phi < O\,(\varepsilon^2),\ O\,(\varepsilon^{-1}) < \eta - \eta_1^+\}$ where only the normal transfer is important will not be considered. We only indicate the order (with respect to ε) of concentration in these regions: $c_1^{(2)} \sim \sqrt{\varepsilon}$, $c_1^{(3)} \sim \varepsilon$, and $c_1^{(4)} \sim 1$. A detailed analysis of these regions in an axisymmetric flow appears in [1, 4—6]. The more complex analysis of a three-dimensional diffusion boundary layer was carried out in [7], where a transformation was obtained for reducing the equation of a steady convective diffusion, expressed in terms of boundary layer coordinates, to an equation with separable variables.

3. Diffusion to an arbitrary particle of the array. It is assumed that the distance between particles satisfies the condition $\overline{\eta_{k+1}} - \eta_k^+ < O\,(\varepsilon^{-1})$. Regions of the trailing $W_2^{(3)} = \{\Phi < O\,(\varepsilon^4),\ |\,\eta - \eta_2^+\,| < O\,(\varepsilon)\}$ and leading $b_2 = \{\Phi < O\,(\varepsilon^4),\ |\,\eta - \eta_2^-\,| < O\,(\varepsilon)\}$ stagnation points and, also, the region $d_2 = \{\Phi < O\,(\varepsilon^2),\ O\,(\varepsilon) < \eta - \eta_2^-,\ O\,(\varepsilon) < \eta_1^+ - \eta\}$ of the diffusion boundary layer can be separated on the surface of the second particle. Contributions of regions $W_2^{(3)}$ and b_2 to the total diffusion flux are insignificant in comparison with the contribution of d_2. Hence for the determination of the principal term of the expansion in powers of ε of the total diffusion flux on the second particle, it is sufficient to obtain the solution for the diffusion boundary layer d_2.

The equations and boundary conditions for concentration in the diffusion boundary layer of the second particle are of the form

$$L\,(t_2,\,\zeta)\,c_2^{(d)} = 0,\quad t_2\,(\eta,\,\lambda) = \tau\,(\eta,\,\eta_2^-,\,\lambda) \qquad (3.1)$$

$$c_2^{(d)}\big|_{\zeta \to 0,\ \eta_2^- \leqslant \eta \leqslant \eta_2^+} = 0,\quad c_2^{(d)}\big|_{\zeta \to \infty} \to 1 \quad (\zeta = \varepsilon^{-1}\Phi^{1/2})$$

where τ and L are defined by (2.5).

The conditions of flow-on for the second particle boundary layer are obtained by joining with solutions in regions $W_1^{(1)}$ and $W_1^{(2)}$ of the first particle. Using the solution of Eq. (3.1) with arbitrary initial conditions it is possible to show that the concentration distribution in the inner region of the first particle wake does not affect the leading term of expansion in powers of ε of solution in d_2. Hence the flow-on conditions for the second particle are determined by formula (2.8).

It can be shown by a reasoning similar to that used in the case of the first particle that Eq. (3.1) is not sufficient for determining the concentration distribution in the neighborhood of the second particle stagnation point. Hence it is necessary to consider the diffusion wake W_2 of the second particle whose convection boundary layer region $W_2^{(1)} = \{\Phi < O(\varepsilon^2), O(\varepsilon) < \eta - \eta_2^+\}$ determines the flow-on condition for the diffusion boundary layer of the third particle.

The recurrent system of equations that defines the concentration distribution in the diffusion boundary layer of any particle is of the form

$$L(t_k, \zeta) c_k^{(d)} = 0 \quad (k = 1, 2, \ldots, n) \tag{3.2}$$

$$c_k^{(d)}\big|_{\zeta=0, \eta_k^- \leqslant \eta \leqslant \eta_k^+} = 0, \quad c_k^{(d)}\big|_{\zeta \to \infty} \to 1$$

$$c_k^{(d)}\big|_{t_k=0} = c_{k-1}^{(d)}(\zeta, t_{k-1}^0), \quad c_0^{(d)} = 1$$

$$t_k = \tau(\eta, \eta_k^-, \lambda), \quad t_k^0 = \tau(\eta_k^+, \eta_k^-, \lambda) \quad (\zeta = \varepsilon^{-1}\Phi^{1/2})$$

where τ and L are defined by Eq. (2.5).

Introduction of the new variable

$$t_* = t_*(\eta, \lambda) = \sum_{i=1}^{k-1} t_i^0 + t_k \tag{3.3}$$

reduces system (3.2) to the single equation

$$L(t_*, \zeta) c = 0, \quad c\big|_{t_*=0} = 1$$
$$c\big|_{\zeta=0} = 0, \quad c\big|_{\zeta \to \infty} \to 1 \tag{3.4}$$

whose solution is of the form

$$c = \Gamma^{-1}(1/3)\,\gamma(1/3, \zeta^3/9t_*) \tag{3.5}$$

The local and total diffusion fluxes on the k-th particle are determined by formulas

$$j_k(\eta, \lambda) = \frac{f^{1/2}(\eta, \lambda)}{\varepsilon \sqrt{g_{11}^0}\,\Gamma(1/3)} \frac{3^{1/3}}{t_*^{1/3}} \quad (\eta_k^- \leqslant \eta \leqslant \eta_k^+) \tag{3.6}$$

$$I_k = \int_0^{2\pi} \int_{\eta_k^-}^{\eta_k^+} j_k \Lambda \sqrt{g_{11}^0}\, d\eta\, d\lambda = I_\Sigma^{(k)} - I_\Sigma^{(k-1)}$$

$$I_\Sigma^{(k)} = \varepsilon^{-1}\Gamma^{-1}\left(\frac{1}{3}\right) 3^{1/3} \int_0^{2\pi} \Big[\sum_{i=1}^{k} t_i^0(\lambda)\Big]^{2/3} d\lambda$$

We use formulas (3. 3) and (3. 6) for investigating the total flux on a particle in the array at high numbers k

$$k \to \infty, \quad I_k \to \frac{2 \cdot 3^{1/3}}{\varepsilon \Gamma (1/3)} \int\limits_0^{2\pi} t_k^{\ 0} (\lambda) \Big[\sum_{i=1}^k t_i^{\ 0} (\lambda) \Big]^{-1/3} d\lambda \qquad (3.7)$$

For an array of periodic structure (any hydrodynamic parameter that defines   the array satisfies the condition $\alpha (\eta + T, \lambda) = \alpha (\eta, \lambda), \quad T = \eta_2^- - \eta_1^-)$ from formula (3. 7) we have

$$I_\Sigma^{(k)} = I_1 k^{2/3}, \quad I_k = I_1 [k^{2/3} - (k-1)^{2/3}] \qquad (3.8)$$

$$k \to \infty, \quad I_k \to {}^2/_3 I_1 k^{-1/3}, \quad I_1 = \varepsilon^{-1} \Gamma^{-1} \Big(\frac{1}{3} \Big) 3^{1/3} \int\limits_0^{2\pi} t_1^{2/3} (\lambda) \, d\lambda$$

where I_1 is the total diffusion flux on the first particle.

Let us now consider the flow field which defines, besides isolated stagnation points, the critical lines at the particle surfaces. Such lines determine the two-dimensional carrying surfaces which may be investigated as in [2], where it was shown that in that case the diffusion wake consists only of the region of the trailing (flow-off) stagnation point and of the mixing region of an over-all dimension $O (P^{-1/6})$. This indicates that when the dimensions of the closed circular region (which develops behind particles at Reynolds numbers $R \infty 10$) the diffusion interaction in the array is negligible, and the total stream of matter to every particle can be determined exclusively by the local velocity filed in the proximity of its surface.

We shall explain this on the example of an array in a flow field of periodic struc - ture. We assume that the flow field in the vicinity of a particular particle depends only on some parameter ω (e. g., the Reynolds Number), and with increasing ω behaves as follows: for $0 \leqslant \omega \leqslant \omega_1$ there are only two isolated stagnation points, when $\omega_1 < \omega$ there appear downstream of particles closed circulation regions whose dim- ensions increase with increasing ω, and when $\omega_2 < \omega$ the characteristic dimen - sions of the closed circulation region become considerably greater than $P^{-1/6}$.

Let us investigate the dependence of mass transfer in such system on parameter ω. Formulas (3. 8) are valid for the total diffusion flux on the particle when $0 \leqslant \omega \leqslant \omega_1$. A region of closed circulation appears downstream of particles when $\omega_1 < \omega$, the diffusion wake pattern begins to change, and for $\omega_2 < \omega$ it consists only of the trailing stagnation point and of the mixing regions. The diffusion interaction between particles can then be neglected, and the total diffusion fluxes determined by formula

$$I_k = I_1, \quad I_\Sigma^{(k)} = k I_1$$

When $\omega_1 < \omega < \omega_2$ the effect of inter-particle diffusion requires separate in- vestigation; it is, however, possible to assume that formula

$$I_\Sigma^{(k)} = I_1 (\omega) \, k^{\gamma(\omega)}, \quad {}^2/_3 \leqslant \gamma (\omega) \leqslant 1 \qquad (3.9)$$

is valid for the over-all diffusion flux throughout the range of parameter ω variation.

Note that all formulas derived in Sects. 2 and 3 are valid for any system of coor - dinates $(\xi^*, \eta^*, \lambda^*)$ equivalent to (ξ, η, λ) (i. e. $e_\xi = e_{\xi^*}, \ e_\eta = e_{\eta^*}, \ e_\lambda = $

Θ_{λ^*}, $g_{kk} / g_{kk}{}^* = O(1)$, $k = 1, 2, 3)$.

The axisymmetric case. In this case $\partial / \partial\lambda = 0$ and the coordinate system introduced in Sect. 1 in the proximity of particle surface is converted into a conventional coordinate system which is often used in the analysis of the hydrodynamic boundary layer.

For the complete diffusion flux formula (3.6) is simplified, assuming the form

$$I_\Sigma^{(k)} = 2\pi e^{-1} 3^{4/3}\Gamma^{-1}\left(\frac{1}{3}\right)\left[\sum_{i=1}^{k} t_i^0\right]^{2/3}, \quad \Lambda(\eta) = \left[\sqrt{g}\,g_{11}^{-1}\right]_{\xi=0} \tag{3.10}$$

$$t_k^0 = 2^{-5/2} \int_{\eta_k^-}^{\eta_k^+} \left[\Lambda\left(\frac{\partial^2\psi}{\partial\xi^2}\right)^{1/2}\right]_{\xi=0} d\eta$$

We shall now consider an array of spheres of radius $\varkappa(k) = O(1)$ spaced at distance $1 \ll l_k \ll \varepsilon^{-1}$ one after the other along the axis of an advancing Stokes stream. The radius of the first sphere is taken as the characteristic dimension of particles. In a spherical system of coordinates fixed at the center of the k-th particle the stream function is defined by

$$\psi_k = {}^3/_4 \, (r_k - \varkappa(k))^2 \sin^2 \theta_k + O(l_*^{-1}) + O(r_k - \varkappa(k))^3 \tag{3.11}$$
$$l_* = \min_k l_k$$

Taking into account that in proximity of a sphere $\Lambda = \varkappa^2(k) \sin \theta_k$, using formulas (3.10) and (3.11) for the total diffusion fluxes we obtain

$$I_\Sigma^{(k)} = I_1\Big[\sum_{i=1}^{k} \varkappa^2(i)\Big]^{2/3}, \quad I_k = I_\Sigma^{(k)} - I_\Sigma^{(k-1)} \tag{3.12}$$

$$k \to \infty, \quad I_k \to \frac{2}{3}\varkappa^2(k)\Big[\sum_{i=1}^{k} \varkappa^2(i)\Big]^{-1/3} I_1$$

$$I_1 = (3\pi)^{2/3} / (2\varepsilon\Gamma(1/3))$$

Setting in (3.12) $\varkappa(k) = 1$ we obtain the results derived in [1].
When $\varkappa(k) = k^\nu$ formulas (3.12) yield for the diffusion fluxes the expressions

$$k \to \infty, \quad I_\Sigma^{(k)} \to (2\nu + 1)^{-2/3} k^{(4\nu+2)/3} I_1 \tag{3.13}$$

$$I_k \to {}^2/_3 (2\nu + 1)^{1/3} k^{(4\nu-1)/3} I_1 \quad (\nu > -1/2)$$

Let us determine the law of growth of the radii of spheres $\varkappa(k)$ in the array for which the diffusion fluxes on each of these are equal, i.e. $I_k = I_1$. Using (3.12) and taking into account that $I_\Sigma^{(k)} = kI_1$ for the sphere radii we have

$$\varkappa(k) = \sqrt{k^{3/2} - (k-1)^{3/2}}, \quad \varkappa(k \to \infty) \to \sqrt{{}^3/_2}\,k^{1/4} \tag{3.14}$$

4. The plane case. Let us consider the problem of diffusion to an array of cylinders in a laminar stream of viscous incompressible fluid. We assume that the stream

function is known from the solution of the related problem of flow past the array and that each cylinder has only two stagnation points. The array divides the whole flow region into two subregions in which the stream function sign is constant.

We introduce the coordinates ξ and η, and use the same numbering of bodies and notation as in Sects. 1 and 2. The concentration distribution in the stream is determined by the solution of Eq. (2.3) taking into account that $\partial / \partial \lambda = 0$ and $g_{33} = 1$. The stream function in proximity of the array can be represented as

$$\xi \to 0, \quad \psi(\xi, \eta) \to \xi^2 f(\eta) \quad (\eta_k^- < \eta < \eta_k^+) \tag{4.1}$$
$$\xi \to 0, \quad \psi(\xi, \eta) \to \xi h(\eta) \quad (\eta_k^+ < \eta < \eta_{k+1}^-)$$

In what follows we consider, without loss of generality, the region $\psi \geqslant 0$.

The asymptotic analysis of problem (2.3), (4.1) shows that (when $\varepsilon \to 0$) in proximity of cylinder surfaces there are four regions with different mass transfer mechanisms [2]. These are: the outer region e, region $d_k = \{\xi < O(\varepsilon), \eta_k^+ - \eta < O(\varepsilon), \eta - \eta_k^- < O(\varepsilon)\}$ of the diffusion boundary layer, region $W_k^{(3)} = \{\xi < O(\varepsilon), |\eta - \eta_k^+| < O(\varepsilon)\}$ of the trailing stagnation point, and the mixing region $W_k^{(4)} = \{\psi < O(\varepsilon^2), O(\varepsilon) < \eta - \eta_k^+ < O(\varepsilon^{1/3})\}$. The convective boundary layer and the inner region of the diffusion wake are in this case absent, and the over-all length of the diffusion wake is of the order of $\varepsilon^{1/3}$. Hence when the distance between cylinders is considerably greater than $P^{-1/3}$, the total diffusion flux to any cylinder is determined only by the local flow field near its surface.

The concentration distribution in the diffusion boundary layer of the k-th cylinder is determined by the solution of Eq. (2.5) with the initial condition specified by the concentration in the diffusion wake of the preceding $(k-1)$-st cylinder. For the first cylinder we have $c_1^{(d)}(t_1 = 0) = 1$.

Direct interaction between the diffusion boundary layers of cylinders occurs when $l_k = \eta_{k+1}^- - \eta_k^+ < O(\varepsilon)$, and it is then possible to derive a recurrent system of equations similar to (3.2). It is, consequently, possible to use formulas (3.5) and (3.6) for determining the concentration and local diffusion flux, taking into account that $g_{33} = 1$. For the total diffusion flux (to the part of the cylinder adjacent to region $\psi \geqslant 0$) we obtain

$$I_\Sigma^{(k)} = \varepsilon^{-1} 3^{4/3} \Gamma^{-1}\left(\frac{1}{3}\right) \left[\sum_{i=1}^k t_i^0\right]^{2/3}, \quad I_k = I_\Sigma^{(k)} - I_\Sigma^{(k-1)} \tag{4.2}$$

$$t_i^0 = \frac{1}{2} \int_{\eta_i^-}^{\eta_i^+} f^{1/2} \Lambda \, d\eta, \quad \Lambda = \left[\sqrt{\frac{g_{22}}{g_{11}}}\right]_{\xi=0}$$

In the case of an array of cylinders of periodic structure formulas for the total diffusion fluxes are determined by formulas (3.8), where I_1 is the total diffusion flux to the first cylinder. For a bundle of cylinders consisting of arrays of circular cylinders of the same radius arranged on a common axis at distance $l < O(\varepsilon)$ from each other we have

$$I_1 = \frac{\varepsilon^{-1} 3^{4/3} (2\pi\Omega)^{1/3}}{\Gamma(^1/_3)} \left(\frac{\Gamma(^3/_4)}{\Gamma(^5/_4)}\right)^{2/3}, \quad f(\theta) = \Omega \sin \theta$$

where Ω is a constant dependent on the bundle porosity and different in different cellular models (see, e.g., [9,10]).

For $O(\varepsilon) < l_k < O(\varepsilon^{1/2})$ the condition of flow-on on the k-th cylinder is determined by the concentration in the mixing region of the $(k-1)$-st cylinder. Below we assume for simplicity that the singular streamline lies on the axis of symmetry of the stream. By the substitution

$$x_k = x_k(\eta) = \varepsilon^{-1} \int_{\eta_k^+}^{\eta} h(\eta)\Lambda(\eta)\,d\eta, \quad z = \varepsilon^{-2}\psi \quad (z = \zeta^2)$$

the problem of concentration distribution in the mixing region is reduced to the conventional equation of thermal conductivity [2]

$$\left(\frac{\partial}{\partial x_k} - \frac{\partial^2}{\partial z^2}\right)c_k^{(4)} = 0, \quad \left.\frac{\partial c_k^{(4)}}{\partial z}\right|_{z=0} = 0, \quad \left.c_k^{(4)}\right|_{z\to\infty} \to 1 \qquad (4.3)$$

with the initial condition determined by the concentration at the boundary of the diffusion boundary layer d_k.

Using the solution of Eqs. (2.5) and (4.3) woth arbitrary initial condition, for the concentration in the diffusion boundary layer of any of the cylinders we obtain the formula

$$c_k^{(d)}(\zeta, t_k) = A(\zeta, t_k) * c_{k-1}^{(4)}(z, \overset{\bullet}{x}_{k-1}), \quad c_0^{(4)} = 1 \quad (z = \zeta^2)$$
$$c_k^{(4)}(z, x_k) = B(z, x_k) * c_k^{(d)}(\zeta, t_k^\circ), \quad x_k^\circ = x_k(\eta_{k+1}^-) \qquad (4.4)$$

$$A(\zeta,\tau) * u(\zeta) = \int_0^\infty \zeta^* \frac{(\zeta\zeta^*)^{1/2}}{3\tau} \exp\left(-\frac{\zeta^3 + \zeta^{*3}}{9\tau}\right) I_{1/3}\left(\frac{2\zeta^{3/2}\zeta^{*3/2}}{9\tau}\right) \times$$
$$u(\zeta^*)\,d\zeta^*$$

$$B(z,\tau) * u(z) = \int_0^\infty \frac{1}{\sqrt{\pi\tau}} \exp\left(-\frac{z^2 + z^{*2}}{4\tau}\right) \mathrm{ch}\left(\frac{zz^*}{2\tau}\right) u(z^*)\,dz^*$$

It can be shown that formulas (4.4) are valid for any distance between cylinders in the array $0 \leqslant l_* < \infty$.

5. Discussion of results. The diffusion wake mixing region $W_k^{(4)} = \{\Phi < O(\varepsilon^2), \rho_k = \varepsilon(\eta - \eta_k^+) = O(1)\}$ of three-dimensional bodies of finite dimensions can be analyzed as in [1]. The concentration in that region levels out to the undepleted one $(c_k^{(4)} \to 1)$. Hence, if the distance between particles in the array is resonably great $\eta_k^- - \eta_{k-1}^+ > O(\varepsilon^{-1})$, the concentration distribution and the diffusion fluxes on the k-th particles are determined exclusively by the local velocity field near its surface.

The analysis in Sections $1-3$ indicates that when $\Phi \gg \varepsilon^2$ the concentration of the substance dissolved in the stream is virtually the same as in the flow-off stream. Consequently, when the distance between arrays is considerable in comparison with $P^{-1/3}$, the arrays have no effect whatsoever (as regards diffusion) on each other.

The above analysis is in qualitative agreement with experimental data on heat and mass transfer in dispersed systems (see, e.g., [11]). Sections 3 and 4 show that at low Reynolds numbers in concentrated dispersed systems the structure of the singular stream-

lines which commence and end at the surface of particles is the determining factor in the mass transfer between the solid and the fluid phases. In such cases the stream contains a large number of arrays in which the internal mass transfer is considerably slowed down by the interaction of diffusion wakes and boundary layers of particles belonging to an array. For example, the total diffusion flux on particle decreases as $k^{-1/3}$ when $k \to \infty$ (k is the ordinal number of an array particle), while the over-all flux on all array particles is proportional to $k^{2/3}$ and considerably smaller than the over-all flux calculated by the self-similar solution [8] (with no allowance for the diffusion interaction of particles), which makes it proportional to k.

This behavior is maintained with increasing Reynolds number until downstream of particles there are no closed circulation regions that contain a circular vortex. Diffusion waves begin to "blurr" when the closed circulation region is formed and, when the vortex becomes fairly large, their characteristic dimensions are of order $P^{-1/3}$ [2]. When the distance between particles is considerably greater than $P^{-1/3}$, the mass transfer between the solid and fluid phase is additive, i.e. the total diffusion flux on a particle is determined by the local flow field near its surface.

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